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Mass transfer from cylinders rotating in Newtonian fluids and dilute polymer solutions

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A new theoretical expression for tubulent mass transfer from a rotating cylinder has been proposed using Levich's three-zone model. The predictions of this model, which has no adjustable parameters, were compared with available experimental data and with other correlations for Newtonian fluids and the agreement was found very satisfactory. Equally good agreement was found between the predictions of our model and the available data for mass transfer from a rotating cylinder under the conditions of maximum drag reduction.

Nomenclature

A constant in Equation 18
D diffusivity
D_E eddy diffusivity
d diameter of cylinder
f friction factor
h heat transfer coefficient
k mass transfer coefficient
l mixing length
l_c modified mixing length defined by Equation 19
Nu $= hd/\alpha_H$, Nusselt number
Pr $= \nu/\alpha_H$, Prandtl number
Re $= dU/\nu$, Reynolds number
Sc $= \nu/D$, Schmidt number
Sh $= kd/D$, Sherwood number
U velocity at the wall
v_0 friction velocity
v_y' root mean square of y-directional fluctuation velocities
y distance perpendicular from surface
α constant in Equation 4
α_H thermal diffusivity
β value of dimensionless laminar sublayer thickness
δ_1 thickness of laminar sublayer
δ_2 thickness of diffusion sublayer
η fraction in Equation 3
ν kinematic viscosity
ρ density
τ_0 tangential stress exerted on surface

Subscript
D dilute polymer solution

1. Introduction

The turbulent mass transfer from and to a rotating cylinder has found many applications in the electrochemical industry, namely in electrodeposition, coating, dissolution, and in the study of corrosion and of liquid metals. The process has been extensively studied for cylinders rotating in Newtonian solutions [1, 2] since the boundary conditions and the concentration field are the same over the entire surface, if the end effects can be neglected and if the cylinder rotates in an infinite pool of liquid.

The mechanism of turbulent transport in liquids can be profoundly altered by the addition of a small amount of certain soluble polymers known as the 'drag reducing' additives. Elata and Tirosh [3] and Killen and Almo [4] found that the torque of cylinders rotating in solutions containing these drag reducing additives is considerably reduced. Along with this, the turbulent mass transfer rate is also significantly reduced, as it was found by Sedahmed *et al.* [5–11] who correlated their data [9] by using the phenomenological approach of Astarita *et al.* [12].

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In this work, we shall attempt to develop a new model for the turbulent mass transfer from a spinning cylinder based on the 'three-zone' concept of Levich [13] and we shall apply this model to predict the mass transfer rate when the turbulent drag is reduced. Hydrodynamic data rather than mass transfer data are used to determine the constants in the proposed model. In doing this, we shall follow a line of thoughts which we have used earlier [14] in developing a transport model applicable under the conditions of maximum drag reduction.

2. Three-zone model

When the Schmidt number is large (as in the case for liquids), the significant variation of the concentration is confined to a region very close to the wall. It seems reasonable to assume that the conditions very close to the wall are the same at the given friction factor, whether the wall is a rotating cylinder or a round pipe. In fact, Smith and Greif [15] found that, in the first approximation, the expression for heat or mass transfer to a rotating cylinder at large values of the Prandtl or Schmidt number is consistent with the result of Deissler [16] for pipe flows.

We have shown earlier [14] that, for large Sc, the diffusion sublayer thickness δ_2 effectively controls the rate of mass transfer in Newtonian and viscoelastic liquids since

$$k = \frac{D}{1.5\delta_2} \tag{1}$$

In order to estimate the mass transfer coefficient by Equation 1, the value of δ_2 needs to be determined first.

Within the laminar sublayer ($y \leqslant \delta_1$), the eddy diffusivity D_{E} is given by

$$D_{\mathrm{E}} = v_y' l \tag{2}$$

where the fluctuation velocity v_y' may be written as [17]:

$$v_y' = \eta y^2 v_0 / \delta_1^2 \tag{3}$$

Assuming that the mixing length l is proportional to the distance from the wall

$$l = \alpha y \tag{4}$$

and that the dimensionless thickness of the laminar sublayer δ_1^+ is defined as

$$\delta_1^+ \equiv \delta v_0/\nu = \beta, \tag{5}$$

we have

$$D_{\mathrm{E}} = \frac{\alpha\eta}{\beta^2} y^3 v_0^3 \nu^{-2} \tag{6}$$

The thickness of the diffusion sublayer δ_2 is evaluated by putting $D = D_{\mathrm{E}}$ at $y = \delta_2$, whence

$$\delta_2 = \left(\frac{\beta^2}{\alpha\eta}\right)^{1/3} D^{1/3} \nu^{2/3} v_0^{-1} \tag{7}$$

The friction velocity defined by $v_0 = (\tau_0/\rho)^{1/2}$ can be represented by

$$v_0 = U(f/2)^{1/2} \tag{8}$$

Substitution of Equations 7 and 8 into Equation 1 gives

$$Sh = \frac{kd}{D} = 0.67\left(\frac{\beta^2}{\alpha\eta}\right)^{-1/3}\left(\frac{f}{2}\right)^{1/2} Sc^{1/3} Re \tag{9}$$

Eisenberg *et al.* [19] found that the expression for the friction factor of a cylinder rotating in Newtonian fluids put forward by Theodorsen and Regier [18].

$$\frac{1}{f^{1/2}} = -1.825 + 4.07 \log_{10}(f^{1/2} Re) \tag{10}$$

can be approximated by [19]

$$f/2 = 0.079 Re^{-0.30} \tag{11}$$

Substituting Equation 11 into Equation 9 and using the values for α, β, and η proposed by Davies [17] for Newtonian liquids

$$\alpha = 0.4, \quad \beta = 5, \quad \eta = 0.09$$

we obtain an expression for the mass transfer from a rotating cyclinder to a Newtonian liquid in the form

$$Sh = 0.0213 Sc^{1/3} Re^{0.85} \tag{12}$$

In the above derivation, effects of centrifugal forces which act in the same direction as the primary direction of transport have been ignored. Turbulent mixing, however, may be enhanced by centrifugal forces and, therefore, we incorporate this effect following the approach of Smith and Greif [15] by specifying a modified mixing length given as

$$l_c = l\left(1 + \frac{20}{Re} \times \frac{2}{f}\right) \tag{13}$$

This modification increases the mixing length and ultimately the mass transfer rate. Its effect is particularly significant at low Reynolds numbers as it can be seen from Equation 14 which results from combining Equation 12 with Equation 13

$$Sh = 0.0213 Sc^{1/3} Re^{0.85} (1 + 253 Re^{-0.7})^{1/3} \tag{14}$$

In order to examine the effect of drag reduction on the rate of mass transfer we shall need a reliable correlation for the friction factor of a cylinder rotating in dilute polymer solutions. Such a correlation is, however, missing. Therefore, we have to resort to an approximation. Mashelkar [20] re-examined the available data on drag reduction in miscellaneous external rotational flows using the phenomenological approach proposed by Astarita *et al.* [12] and found that the reduced friction factor asymptotically approaches a value of about 0.3. We shall further take into account the measurement of Killen and Almo [4] who found the torque of a cylinder rotating in 50 ppm solution of polyethylene oxide reduced by about 40% without the exponent of the Reynolds number being changed in any significant way. By considering these two results, we assume that the drag coefficient of a rotating cylinder under the condition of maximum drag reduction can be approximated by

$$f_D/2 = 0.30 f/2 \tag{15}$$

Substituting Equation 15 into Equation 9 and making use of the coefficients α_D, β_D, and η_D found previously [14]

$$\alpha_D = \frac{l}{\alpha} = 0.4, \quad \beta_D = 11.6, \quad \eta_D = 0.129$$

we shall obtain an expression for the Sherwood number in the form

$$Sh = 0.00751 Sc^{1/3} Re^{0.85} \tag{16}$$

The interaction between the inertia and the viscoelasticity is, potentially, a more complex matter. Although in unconfined flows (unlike in the case of a Couette flow) instabilities of the type of Taylor vortices have not been reported and, thus, one need not expect the viscoelasticity to interfere with their structure it is, nevertheless, far from clear whether the elastic forces would not lead to the alteration of the primary velocity profile in the close vicinity of the solid boundary as is the case for a spinning disk [21]. This aspect definitely deserves further attention.

In the absence of a more rigorous treatment we shall apply Equation 13 to the present case as well. Thus, after some manipulation, the equation for the mass transfer from/to the surface of a spinning cylinder under the conditions of maximum drag reduction will be

$$Sh = 0.00751 Sc^{1/3} Re^{0.85} (1 + 1690 Re^{-0.7})^{1/3} \tag{17}$$

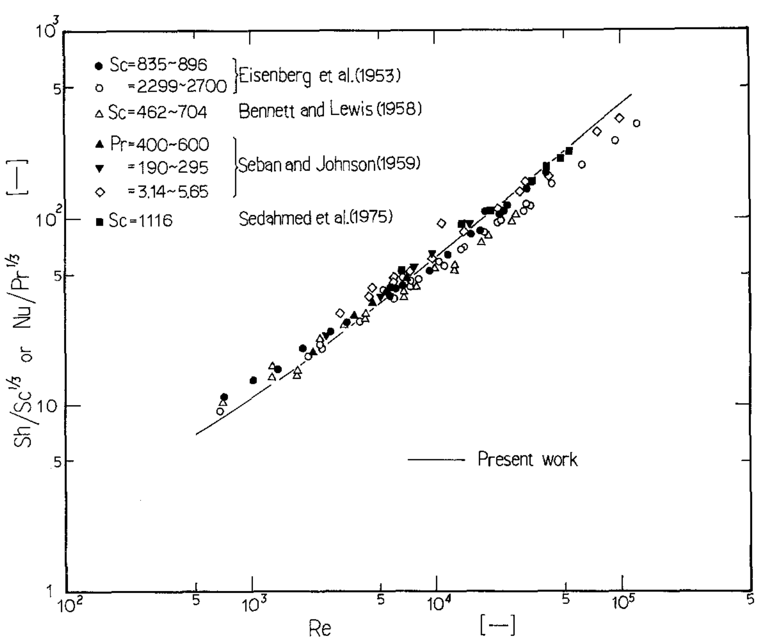

Fig. 1. Comparison of predictions of this model with mass transfer data for Newtonian fluids.

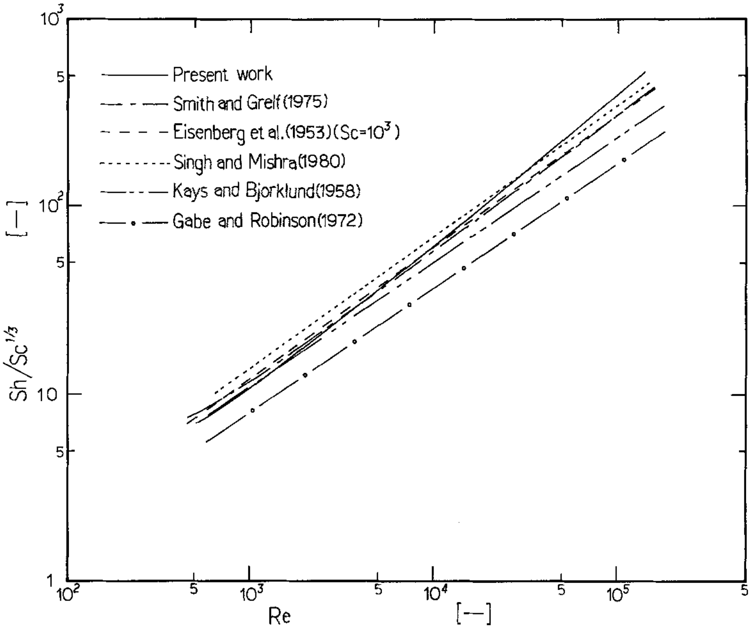

Fig. 2. Comparison of predictions of this model with various correlations for Newtonian fluids.

3. Discussion

First, we test the applicability of the model based on Levich's three-zone concept using the available correlations and experimental data for Newtonian fluids.

Figure 1 compares the experimental data reported by Eisenberg *et al.* [19], Bennett and Lewis [22], Seban and Johnson [23], and Sedahmed *et al.* [5]. It is seen that Equation 14 is in good agreement with the available experimental data. In order to test the predictive capability of our model further we shall compare Equation 14 with other correlations presented in the literature. This comparison is shown in Fig. 2. On the whole, Equation 14 is in good agreement with the correlation based on the Chilton and Colburn analogy (Eisenberg *et al.* [19]), with that based on the modified Prandtl analogy (Singh and Mishra [24]), and with that of Smith and Greif [15]. The line based on the von-Karman type analogy (Kays and Bjorklund [25]) lies below other correlations in the high Reynolds number region (i.e. $Re > 10^4$). Our model expressed by Equation 14 is seen to be in good agreement with the various correlations and experimental data over the range $5 \times 10^2 < Re < 10^6$.

In this context, it may be appropriate to mention the work of Gabe and Robinson [26] who proposed a model based on an assumption of a concentration boundary layer, viscous sublayer and of a region of fully developed turbulence. In deriving the expression for the eddy diffusivity, they took into account the curvature of the solid boundary. The coefficient A in their equation:

$$Sh = A\,Re^{2/3}Sc^{1/3} \tag{18}$$

was evaluated from the mass transfer data and found 0.079. In a later review paper, Gabe [2] stated that the magnitude of A can be found directly from the value of the friction factor as given by Theodorsen and Regier [18] but no substantiation has been offered except that both A and the friction factor happen to have the same numerical value. Although the coefficient A has been determined from experimental data, the match of Equation 18 with the available experimental results is not convincing. As shown in Fig. 2, it predicts a substantially lower Sherwood number. This may well be because Equation 18 ignores the influence of inertia on the mass transfer rate.

A further test of the validity of the present model is provided by its comparison with an empirical correlation and with experimental data obtained with cylinder rotating in dilute polymer solutions. Sedahmed *et al.* [5] and Al-Taweel *et al.* [9] measured mass transfer rates from a rotat-

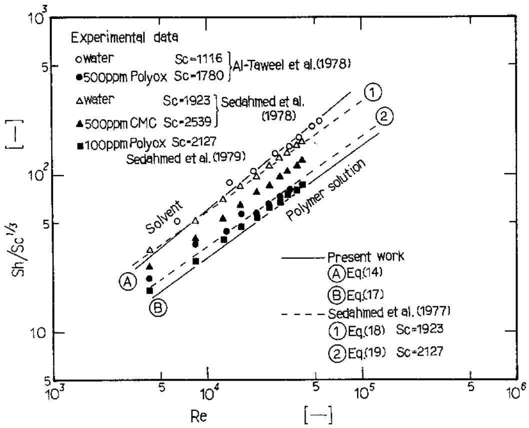

Fig. 3. Comparison of predictions of this model with mass transfer data for dilute polymer solutions.

ing cylinder to drag-reducing polymer solutions of 500 ppm Polyox 301. At this concentration, the reduction of the turbulent drag is almost at the Virk's asymptote for maximum drag reduction. (See Fig. 6 in Reference [9]).

The comparison is shown graphically in Fig. 3. It can again be seen that our model expressed by Equation 17 is in a reasonable agreement with the available experimental data. Sedahmed *et al.* [6] suggested two empirical equations for the mass transfer from rotating cylinders, for water:

$$Sh = 0.079Re^{0.7}Sc^{0.356} \tag{19}$$

for Polyox solution:

$$Sh = 0.0475Re^{0.7}Sc^{0.356} \tag{20}$$

Also these two empirical equations agree well with our model, both in its Newtonian form, Equation 14 as well as for the conditions at maximum drag reduction, Equation 17. Towards lower Reynolds numbers, the intensity of turbulence is lower and the conditions for the maximum drag reduction are not entirely met. This is illustrated in Fig. 3, by a slight deviation of the data from the maximum mass transfer reduction asymptote. Similarly, the deviation from the asymptote is demonstrated by the data points obtained by Sedahmed *et al.* [8] for mass transfer in 500 ppm solutions of carboxymethyl cellulose (CMC). Again solutions of CMC are not known to be viscoelastic and thus do not exhibit any significant capability to reduce the various turbulent transports. Accordingly, the experimental data for CMC lies between those for water and the maximum reduction asymptote.

4. Conclusions

We have presented a new model of turbulent mass transfer from/to a cylinder rotating in Newtonian and drag reducing liquids. The model which is based on the concept of 'three-zones' of Levich agrees very well both with existing experimental data as well as with some empirical correlations.

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